Contests with Foot-Soldiers

Bharat Goel Arijit Sen

In many real-world contests (*political elections / lobbying for public projects*), *contestants try to engage* foot-soldiers (*unemployed youth / local residents*) *to fight for them. Such contests have the following features*: *a significant part of a contestant's foot-soldier compensation is* contingent *upon the contestant winning the contest*, *and foot-soldiers are* (*at least partially*) mercenary *in that higher compensation offers do induce them to* switch allegiance. *We study a class of contests with the above features*, *where two contestants – a* favourite *and an* underdog *– recruit foot-soldiers by offering contingent* (*and non-contingent*) *compensations in cash or in excludable public goods like political access. Our analysis delineates contest equilibria with the following features*: *Contestants' offers of contingent compensations force potential foot-soldiers to choose their allegiance on the basis of* predicted winners *– and that act*, *in and of itself*, *enables the* favourite *to extend her lead. In some cases*, *it is possible that the* underdog *is doubly disadvantaged – her total compensation bill is no less than the* favourite'*s though she manages to attract a smaller army of foot-soldiers and thus falls farther behind in the race. The contest is necessarily* dissipative *for the underdog*: *she would be strictly better off under a ban on the hiring of foot-soldiers* (*though she is the one who offers higher foot-soldier compensation*). *In some cases*, *the availability of unemployed youth to act as foot-soldiers in contests causes everyone in the economy to lose* (*except*, *maybe*, *the foot-soldiers themselves*).

Keywords: Contests, foot-soldiers, elections, lobbying.

1. Introduction

The following kinds of stories about bilateral contests motivate our paper:

Story 1: Two firms contest to secure an 'indivisible public project' (e.g., a government contract to build a bridge) in a locality. The terms of public contracting are as follows: first the firms will submit 'technical bids', and then an 'expert committee of bureaucrats' will deliberate and determine which firm gets the contract. After the bids are submitted and before the committee begins its deliberations, each firm can recruit local residents as their foot-soldiers. The foot-soldiers of each firm will approach the expert committee members and try to convince them to award the contract to that specific firm. The larger the number of foot-soldiers a firm can recruit *vis-a-vis* its competitor, the greater will be the chance that the firm will eventually win the contract (after taking into account the relative merits of the two firms' initial technical bids).

Current (*incomplete*) *draft*: June 2017. *Authors' affiliation*: Indian Institute of Management Calcutta.

Story 2: Two candidates contest to get elected to a public office in a locality. After their leadership qualities and policy positions are assessed by the voters, the candidates can hire local unemployed youth to act as their foot-soldiers. The foot-soldiers of each candidate will then work towards influencing the voters to vote for that candidate (by campaigning door-to-door, by organizing rallies, by intimidating, etc.). The larger is a candidate's army of foot-soldiers in relation to her rival's, the greater will be the chance that the candidate will eventually win the election (after taking into account the relative merits and policy positions of the two candidates).

Our paper aims to model and study the strategic interactions inherent in these kinds of *contests with foot-soldiers*. We recognize the following two important features of such contests: (*a*) for each contestant's foot-soldiers, a significant part of the compensation that they receive for their efforts in working for the contestant is *contingent* upon the contestant winning the contest, and (*b*) potential foot-soldiers are (at least partially) *mercenary* in that a higher compensation offer from a particular contestant induce them to *switch allegiance*.

We focus on contests incorporating the above features, where two contestants – a *favourite* and an *underdog* – recruit foot-soldiers by offering contingent (as well as non-contingent) compensations in cash or excludable public goods.[1] In our analysis, we address the following issues: Who between the *favourite* and the *underdog* offers larger compensations to the foot-soldiers, and who manages to recruit the bigger foot-soldier army? Does the practice of recruiting foot-soldiers help the front-runner to *extend her lead*, or does it help the underdog to *catch up*?[2] Will any contestant prefer an *exogenous ban* on foot-soldier recruitment? Might both do?

We study a class of contests models that differ with respect to the following issues: whether contestants can offer only non-contingent offers, or only contingent offers, or a mix of the two;[3] and whether foot-soldier compensations are offered *via private goods* (like cash) or *via* an *excludable public good* (like political access). Our analysis delineates the following set of robust features of contest equilibria. Contestants' offer of contingent compensations force potential foot-

[1] To the extent that our analysis studies elections contests, this paper is tangentially related to the literature that models election campaigns as contests; see, for example, Baron (1994), Grossman and Helpman (1996), Prat (2002), Coate (2004), and Dekel *et. al.* (2008). For studies of various kinds of 'unethical practices' in election contests (including the use of foot-soldiers) see Chaturvedi (2005), Collier (2009), and Collier and Vicente (2012).

[2] This issue is related to the broader question of "allocative effects of contests"; see Corchón (2000).

[3] Note that the first case is likely to arise when each contestant is *ex ante* budget-constrained, and can only afford to "pay" her foot-soldiers after she wins the prize. In contrast, the second case is likely to arise when the contestants have sufficient resources up front and lack any credibility of "keeping campaign promises".

soldiers choose their allegiance on the basis of *predicted winners* – and that act, in and of itself, enables the *ex ante favourite* to extend her lead at the cost of the *ex ante underdog*. Specifically, the following situation arises in many of our contest equilibria: the *favourite*'s compensations offers are no greater (and sometimes less) than the *underdog*'s, but still the *favourite* manages to recruit a larger army of foot-soldiers than the *underdog* (and thereby extends her lead). Further, in some cases the underdog is *doubly disadvantaged* – she foots a higher foot-soldier bill while falling farther behind in the race. In every version of our contest model, *contesting with foot-soldiers* is necessarily *dissipative* for the *ex ante underdog*: she would be strictly better off under an externally-imposed ban on foot-soldier recruitment. In contrast, the *favourite*'s equilibrium payoff from *contesting with foot-soldiers* can, in some situations (but not always), be higher than what it would be if hiring foot-soldiers was banned.

With regard to the welfare effect of such contests on the economy at large, our presumption is that (some of) the actions of the foot-soldiers as well as a part of the compensation paid to them (especially compensations in terms of public goods like political access) are detrimental to society. In that vein, we identify contest equilibria in which *all agents* in an economy (except, maybe, the foot-soldiers themselves) would gain from an exogenously-imposed ban on the use of foot-soldiers. Another specific negative effect of the possibility of *contesting with foot-soldiers* that we highlight is the following: If the contestants were disallowed from using foot-soldiers, then it is possible that they would have 'competed in alternative ways' that would actually be beneficial to society. We identify conditions under which the possibility of contesting *via* foot-soldiers depresses socially beneficial *ex ante* investments that the contestants would have otherwise made.

In this context, an underlying theme of our analysis is the following. In the real-world (as also in our model), what allows contestants to recruit foot-soldiers is the presence of a mass of unemployed and under-employed agents in the economy who have negligible opportunity cost of becoming someone's foot-soldier. Thus, the negative social effects of *contesting with foot-soldiers* that we identify can be considered to arise from the lack of economic development, which causes such *foot-soldier banks* to exist in the first place.

The rest of the current draft is organized as follows. Section 2 presents our basic contest model. Equilibrium characterization results for the 'only non-contingent compensations' model, the 'only contingent compensations' model, and the 'contingent and non-contingent compensations' model are presented in Sections 3, 4, and 5 respectively. Section 6 briefly studies a two-stage contest, with *ex ante* investments by the contestants. Some comments about the robustness of our results in contained in the concluding Section 7. An appendix presents a 'brief micro-foundation' of our posited 'contest win probabilities'. Formal proofs will be presented in a

second (as yet unprocessed) appendix.

2. A Model of Bilateral Contest with Foot-soldiers

There are two contestants – L and R – fighting for a prize; they are located at positions 0 and 1 respectively on a line of unit length. A continuum of agents of unit measure is located on the line interval [0, 1]. The contestants compete by making contingent and/or non-contingent compensation offers to the agents to entice them to become foot-soldiers in the contestants' fight for the prize. Each contestant's compensation is targeted to those foot-soldiers that work for her.

First, a publicly observed variable $\theta \in [-½, ½]$ is realized and observed by all; the realized θ is the '*ex ante* difference in win probabilities' of the two contestants. Then, defining $p_i(\theta)$ to be contestant i's *initial win probability* (for $i = L, R$), we have:

$$p_L(\theta) = 0.5(1+\theta) \in [0.25, 0.75], \text{ and } p_R(\theta) = 0.5(1-\theta) \in [0.25, 0.75].$$ [4]

Here, if $\theta < 0$ (resp., $\theta > 0$) then contestant L (respectively, R) will be the *ex ante underdog* while contestant R (respectively, L) will be the *ex ante favourite*. When $\theta \neq 0$, we will identify the *ex ante underdog* as contestant U and the *ex ante favourite* as contestant F.

After θ is publicly observed, each contestant i (for $i = L, R$), in her attempt to recruit foot-soldiers for her cause, announces a non-contingent compensation offer k_i from a feasible set $[0, k^+]$ and/or a contingent compensation offer c_i from a feasible set $[0, c^+]$, where $c^+ \geq 0$, $k^+ \geq 0$ and $(c^+ + k^+) > 0$.[5] As the term suggests, a foot-soldier for any contestant i will receive the announced *contingent* compensation c_i if and only if i wins the contest; we assume that each contestant's contingent offer c_i will be credible due to reputational concerns.

An agent s, located at position $s \in [0, 1]$, will decide on becoming a contestant's foot-solder by making the following cost-benefit analysis.[6] Having observed θ and the offered compensations $\{(c_L, k_L), (c_R, k_R)\}$, the agent will form *updated belief* $\{\pi_L, \pi_R \equiv (1-\pi_L)\}$ about the contestants'

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[4] In Section 6, we will discuss an extension of our model in which θ is determined in a *prior contest* between L and R.

[5] The compensation caps (c^+, k^+) can arise from budget constraints. More importantly, if offering foot-soldier compensations is a *quasi-legal* activity, the bounds of legality can lead to such caps; similarly, if foot-soldier compensations can be made only through *disguised transfer mechanisms* (as emphasized by Tullock (1983)), such disguises might necessitate compensation caps.

[6] In our model, becoming or not becoming a foot-soldier is a {0-1} choice for each agent s; we do not model the more complicated case where each foot-soldier has to decide about 'how hard' to work for a contestant. Our foot-soldiers are *atomistic* in that sense.

chances of winning; we assume that the beliefs will be identical across all agents. We define the *distance* $D(s, i)$ between agent $s \in [0, 1]$ and contestant i as follows: $D(s, L) = s$, and $D(s, R) = 1 - s$.

Given $\{(c_L, k_L), (c_R, k_R)\}$ and $\{\pi_L, \pi_R\}$, if agent s becomes the foot-soldier of contestant i (for $i = L, R$), then his (expected) payoff $w_s(.)$ will be:

$$w_s(c_i, k_i \mid \pi_i) = \{k_i + \delta.\pi_i.c_i + \alpha.[1 - D(s, i)]\}.$$

The first term in $w_s(c_i, k_i \mid \pi_i)$ is the agent's utility from the non-contingent compensation offered by contestant i, while the second term constitutes the expected discounted utility derived by the agent from contestant i's contingent offer, with $\delta \in (0, 1]$ being the common discount factor of all agents.[7] The third term in $w_s(c_i, k_i \mid \pi_i)$ incorporates the following feature: each agent s gets a non-pecuniary benefit from becoming a foot-soldier of a specific contestant i, and the magnitude of this benefit, parameterized by the *allegiance value* $\alpha > 0$, depends on the *closeness* of agent s to contestant i.[8] Given $\{(c_L, k_L), (c_R, k_R)\}$ and $\{\pi_L, \pi_R\}$, agent s will choose to become a foot-soldier of some contestant i only if $w_s(c_i, k_i \mid \pi_i) \geq w_s(c_j, k_j \mid \pi_j)$ for $i, j = L, R$, and $i \neq j$.[9]

We maintain the following parameter restriction for the rest of our analysis:

PARAMETER RESTRICTION [I]: $\alpha > \{k^+ + \delta.c^+\}$.

Parameter restriction [I] implies that for any feasible compensation vector $\{(c_L, k_L), (c_R, k_R)\}$, an agent located at $s = 0$ (respectively, $s = 1$) will strictly prefer to be the foot-soldier of contestant L (respectively, contestant R).[10]

If $\{S_L, S_R\} \in [0, 1] \times [0, 1]$ is the vector of the contestants' *army sizes* (i.e., their measures of foot-soldiers), we posit that the '*ex post* difference in win probabilities' between them will be:

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[7] The compensation variables $\{c_i, k_i\}$ can be taken to be cash or public good transfers, in which case the utility specification supposes that agents are risk-neutral. Alternatively, in some cases, $\{c_i, k_i\}$ can represent contingent and non-contingent 'utility transfers' to (risk-neutral or risk-averse) agents.

[8] The specified 'linear allegiance function' $\alpha.[1 - D(.)]$ can be generalized to a class of non-linear allegiance functions that includes the linear-quadratic form: $A(1 - D(.)) = \alpha_1.(1 - D(.)) + \alpha_2.(1 - D(.))^2$ with $[\alpha_1 + \alpha_2] > 0$ and $[\alpha_1 \times \alpha_2] > 0$.

[9] We are implicitly assuming that each agent has *zero* opportunity cost of becoming the foot-soldier of some contestant, and so every agent will become someone's foot-soldier in every contest equilibrium. In contrast, Goel (2017) studies a model specification in which each soldier has a significant opportunity cost of becoming a foot-soldier, thereby ensuring that agents located around $s = ½$ will not become any contestant's foot-soldier. Most of our results carry over to that model specification as well.

[10] Parameter restriction [I] ensures that for any feasible compensation vector $\{(c_L, k_L), (c_R, k_R)\}$ a unique set of agents will become foot-soldiers of each of the contestants. If we had adopted the linear-quadratic allegiance function (as in footnote 8) then the required restriction would be $[\alpha_1 + \alpha_2] > [k^+ + \delta.c^+]$.

$[\theta + \phi.(S_L - S_R)]$. Here $\phi \in (0, ½)$ measures the effectiveness of foot-soldiers in raising win probabilities. Then, defining $P_i(S_i, S_j \mid \theta)$ to be contestant i's *final win probability*, we have:

$$P_i(S_i, S_j \mid \theta) = \{p_i(\theta) + 0.5\phi.(S_i - S_j)\} \in (0, 1) \text{ for } i, j = L, R, \text{ and } i \neq j.$$ [11]

If $P_i(S_i, S_j \mid \theta) < ½ < P_j(S_j, S_i \mid \theta)$, then we will refer to contestant i as the *ex post underdog* and contestant j as the *ex post favourite*. Given the parameters of the model $\{\theta, \alpha, \delta, \phi\}$ and the contestants' compensation offers $\{(c_L, k_L), (c_R, k_R)\}$, $\{S_L^*(c_L, k_L \mid c_R, k_R), S_R^*(c_R, k_R \mid c_L, k_L)\}$ will constitute *rational-expectations equilibrium army sizes* and $\{P_L^*(c_L, k_L \mid c_R, k_R), P_R^*(c_R, k_R \mid c_L, k_L)\}$ will be the corresponding *equilibrium final win probabilities* of the two candidates, if and only if each agent makes his 'foot-soldiering choice' based on the *correctly anticipated* win probabilities: $\pi_i = P_i^*(c_i, k_i \mid c_j, k_j) \equiv \{p_i(\theta) + 0.5\phi.[S_i^*(.) - S_j^*(.)]\}$ for $i, j = L, R$, and $i \neq j$.

The following proposition characterizes the unique rational-expectations equilibrium for any feasible compensation vector $\{(c_L, k_L), (c_R, k_R)\}$ when parameter restriction [I] holds.

PROPOSITION 1. There exists a unique $s^*(c_L, k_L; c_R, k_R) \in (0, 1)$ such that in the unique rational-expectations equilibrium, all agents $s \in [0, s^*(.)]$ become contestant L's foot-soldiers while all agents $s \in [s^*(.), 1]$ become contestant R's foot-soldiers; $s^*(.)$ varies continuously in compensations. For any offered compensation vector $\{(c_L, k_L), (c_R, k_R)\}$, the equilibrium measure of foot-soldiers joining each candidate i (for $i, j = L, R$, and $i \neq j$) is given by:

$$S_i^*(c_i, k_i \mid c_j, k_j) = 0.5\{1 + [\delta\{p_i(\theta).c_i - p_j(\theta).c_j\} + \{k_i - k_j\}] / [\alpha - 0.5\phi.\delta.(c_i + c_j)]\},$$

and the equilibrium final win probability of each contestant i is given by:

$$P_i^*(c_i, k_i \mid c_j, k_j) = p_i(\theta) + \{0.5\phi[[\delta\{p_i(\theta).c_i - p_j(\theta).c_j\} + \{k_i - k_j\}] / [\alpha - 0.5\phi.\delta.(c_i + c_j)]\}.$$

Given Proposition 1, we define the contestants' payoffs as follows. For a compensation vector $\{(c_L, k_L), (c_R, k_R)\} \in [0, c^+] \times [0, k^+] \times [0, c^+] \times [0, k^+]$, if player i wins the contest then her payoff will be $\{[V - X(c_i, S_i^*(.))] - Y(k_i, S_i^*(.))\}$, where V is a positive constant representing the *value of the prize* to the contestants, and $X(.)$ and $Y(.)$ are *cost functions* that are non-decreasing in each argument. If player i loses the contest her payoff will be $[-Y(k_i, S_i^*(.))]$.

In their bilateral contest, each contestant i simultaneously announces (c_i, k_i) to maximize her expected utility $EW_i(c_i, k_i \mid c_j, k_j)$ given by:

$$EW_i(c_i, k_i \mid c_j, k_j) \equiv \{P_i^*(c_i, k_i \mid c_j, k_j) \times [V - X(c_i, S_i^*(c_i, k_i \mid c_j, k_j))]\} - \{Y(k_i, S_i^*(c_i, k_i \mid c_j, k_j))\}.$$

[11] This *difference-form* 'impact of foot-soldiering' specification can be generalized in some specific ways. The possible generalizations are discussed in the concluding Section 7.

Here, a compensation vector $\{(c_L^*, k_L^*), (c_R^*, k_R^*)\}$ will constitute a pure-strategy Nash equilibrium of the contest if and only if (c_L^*, k_L^*) is a *best-response* to (c_R^*, k_R^*) and *vice-versa.*[12]

In what follows, we characterize *contest equilibria* for different structures of the contestants' strategy sets and payoff functions. In all cases, we focus exclusively on pure-strategy Nash equilibria. Further, we intentionally specify that the contestants L and R are identical in all respects (especially with respect to their valuations of the contest prize and costs of contesting) except for their initial win probabilities.[13] In doing so, we aim to delineate the impact of asymmetry in initial win probabilities on the behaviour of otherwise similar contestants.[14]

We conclude this section by commenting on the probability vector $\{p_i(.), P_i^*(.)\}$ for $i = L, R$. As stated above, we interpret this vector to be the 'initial and final win probabilities' of contestant i. In the context of an election between two candidates (our 'Story 2' above), these probabilities can be taken as the predictions of *unbiased opinion polls* – one conducted before the time at which the candidates start recruiting foot-soldiers and another conducted just before the elections – that announce the current probability that candidate i will win the election (by garnering a majority of the votes). With regard to a contest between two firms to secure a government contract, we elaborate on our 'Story 1' in the appendix where we provide a simple micro-foundation for the structure of the probability vector $\{p_i(.), P_i^*(.)\}$.

3. Contest Equilibria with only Non-contingent Compensations

In this section, we study the properties of *contest equilibria* in the simplest case where the contestants are restricted to offering only non-contingent compensations to foot-soldiers. [Contestants might be so restricted due to the lack of reputational concerns, thereby making promises of contingent compensations non-credible.] In this case, where $k^+ > 0$ while $c^+ = 0$, each contestant i will simultaneously announce k_i to maximize:

$$EW_i(k_i \mid k_j) \equiv P_i^*(k_i \mid k_j) \times [V] - Y(k_i, S_i^*(k_i \mid k_j)),$$

$$\text{where } P_i^*(k_i \mid k_j) = p_i(\theta) + 0.5(\phi / \alpha)[k_i - k_j], \text{ and } S_i^*(k_i \mid k_j) = 0.5 + 0.5(1/\alpha)[k_i - k_j].$$

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[12] Note that if parameter restriction [I] does not hold, then it is not guaranteed that $EW_i(c_i, k_i \mid c_j, k_j)$ will be a well-defined, continuous, differentiable function of its arguments. Given that, and as we indicated before, we will maintain parameter restriction [I] for the rest of our analysis.

[13] The foot-soldiers are also identical in all respects except for their allegiance to the contestants (which depends *symmetrically* upon the distance between an agent and each consistent).

[14] In the concluding Section 7, we comment on the structure of contest equilibria when the contestants are heterogeneous in their valuations of the contest prize.

Note that in this case, the *endogenous contest success function* $P_i^*(k_i \mid k_j)$ is of the simplest difference-form structure; in contrast, whenever $Y_{12}(.,.) > 0$, the *non-contingent cost function* $Y(k_i, S_i^*(k_i \mid k_j))$ is a function of k_i as well as k_j.

We now present an equilibrium characterization result in this scenario of *only non-contingent offers* under the following (general and mild) restriction on the structure of the cost function:

Parameter restriction [Y]: $Y(k, S)$ is strictly increasing and (weakly) convex in each of its arguments, with $Y(0, S) = 0$ and $Y_{kS}(k, S) > 0$ for all $(k, S) \in [0, k^+] \times [0, 1]$.[15]

PROPOSITION 2. Consider the case where $\{k^+ > 0, c^+ = 0\}$ and parameter restriction [Y] holds. In this case the contest is a *supermodular game* with a dominance-solved symmetric equilibrium $k_L^* = k_R^* = k^* \in [0, k^+]$, with $k^* = 0$ if and only if $\{0.5\phi.V \leq \alpha.Y_k(0, 0.5)\}$.[16] In equilibrium, the contestants have equal army sizes $S_L^* = S_R^* = 0.5$, and their final win probabilities are $P_i^* = p_i(\theta)$ for $i = L, R$. Whenever $k^* > 0$ both contestants strictly prefer a ban on the hiring of foot-soldiers.[17]

Before discussing the important features of the above-described contest equilibrium, let us identify the contestant's equilibrium compensation levels when such compensation takes the form of *non-contingent cash transfers*, with contest cost being determined by $[k_i \times S_i]$ for each i.

COROLLARY 3. Consider the case where $\{k^+ > 0, c^+ = 0\}$ and $Y(k, S) = \gamma.k.S$, with $\gamma \geq 1$ for all $(k, S) \in [0, k^+] \times [0, 1]$.[18] Then in equilibrium: $k^* = max\{0, min\{[(\phi.V/\gamma) - \alpha], k^+\}\}$.

Recall the specific questions about contest equilibria that we posed in Section 1. In the following analysis, we answer these questions not only in the context of the simple contest game with only

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[15] The assumption that $X(0, S) = Y(0, S) = 0$ precludes the following possibility: if a contestant offers no foot-soldier compensation, but some agents still work for her out of a sense of allegiance, then that act does not impose a cost on the contestant. Further, the assumption that $Y_{kS}(k, S) > 0$ precludes the non-contingent compensation to be in terms of a *purely non-rival good*. In contrast, when we subsequently study the case of contingent compensations, we *will* allow for the case where the non-contingent compensation is given *via* an excludable public good (in that case we will have $X_S(c, S) = X_{cS}(c, S) = 0$).

[16] Here, $k^* = k^+$ if and only if $\{0.5\phi V \geq \alpha.Y_k(k^+, 0.5) + 0.5Y_S(k^+, 0.5)\}$; and whenever $k^* \in (0, k^+)$, it is implicitly given by: $0.5\phi V - \alpha Y_1(k^*, 0.5) - 0.5Y_2(k^*, 0.5) = 0$.

[17] In all variants of the contest model studied in this paper, the contestants' equilibrium payoffs are ranked as follows (given that they value the prize identically): Whenever $\theta \neq 0$ the *favourite*'s payoff will be strictly greater than that of the *underdog*, while the players' payoffs will be equal when $\theta = 0$.

[18] Here, $\gamma \geq 1$ is each contestant's per-unit cost of raising funds (cost of capital) for making compensation payments. This cost specification satisfies parameter restriction [Y].

non-contingent offers, but also for more general contests involving contingent compensations. In order to do that, we define the following terminology regarding contest outcomes and contest equilibria. A contest outcome in which neither contestant offers a positive compensation (i.e., $c_L = c_R = k_L = k_R = 0$) will be termed the *null outcome*, and a contest equilibrium generating the *null outcome* will be termed a *null equilibrium*. In contrast, an equilibrium in which at least one contestant i offers (respectively, both contestants offer) positive compensation (i.e., $c_i \times k_i > 0$) will be termed a *non-null* (respectively, *positive*) *equilibrium*.

Next, with regard to contest win probabilities, an equilibrium will be said to be *bias-preserving* if and only if, for $i, j = L, R$, and $i \neq j$, $P_i^*(.) > P_j^*(.)$ whenever $p_i(\theta) > p_j(\theta)$. In contrast, a *bias-reversing equilibrium* would be one where $p_i(\theta) > p_j(\theta)$ implies that $P_i^*(.) < P_j^*(.)$. Note that a bias-preserving equilibrium can be one of three kinds: such an equilibrium can be *bias-maintaining* – with the final win probability of each contestant equaling her initial win probability; or it can be *bias-enhancing* – with $P_i^*(.) > p_i(\theta)$ whenever $p_i(\theta) > p_j(\theta)$; or it can be *bias-mitigating* – with $p_i(\theta) > P_i^*(.) \geq ½$ whenever $p_i(\theta) > p_j(\theta)$.

Finally, note the following: (*a*) A *null equilibrium* is necessarily *bias-maintaining*. (*b*) If $\theta \neq 0$ and if the contest equilibrium is *positive* and *bias-enhancing*, then the equilibrium is necessarily *dissipative for the underdog* in that she will be strictly better off if a ban on the recruitment of foot-soldiers is imposed. (*c*) If, on the other hand, the contest equilibrium is *positive* and *bias-maintaining*, then the equilibrium will be necessarily *dissipative for both contestants.*[19]

Given the above-specified terminology, note the following implications of Proposition 2. An important feature of our contest equilibrium is that when all foot-soldier compensations are independent of the contest outcome, the contestants – who can be asymmetric in terms of their initial win probabilities – behave symmetrically and make identical offers to potential foot-soldiers. For these 'common' offers to be *serious* (i.e., positive), the value of the prize (V) and the effectiveness of foot-soldiering (as parameterized by ϕ) must be large relative to the agents' allegiance value α. Note that smaller is the magnitude of α, the more susceptible are the agents to monetary rewards in choosing which of the two contestants' armies to join. It is in this sense that the following result holds in our contest model: the more *mercenary* are the potential foot-soldiers, the more likely is it that they will be actively wooed by the contestants.[20]

The fact that the contestants offer identical compensations (independent of their initial chances of winning), and the fact that foot-soldiers are not directly affected by the contest outcome, implies

[19] Note that a *null equilibrium* cannot be *dissipative* for any contestant, while a *positive equilibrium* will necessarily be *dissipative* for at least one contestant.

[20] As will be shown subsequently, this logic holds in the various distinct versions of our contest model.

that in equilibrium, the agents split themselves equally between the two contestants, leaving the contestants' final chances of winning unaltered. Thus, the equilibrium is *bias-maintaining*; and further, if the equilibrium is also *positive* then it is *dissipative for both contestants*.

Treating this scenario of 'only non-contingent compensations' as a benchmark case, in the next two sections we study how the contest equilibrium features change when the contestants can make contingent compensation offers. In that regard, our aim to identify contest structures for which every equilibrium will be *bias-enhancing* and *dissipative for the underdog.*

4. Contest Equilibria with only Contingent Compensations

We now turn to the case where the two contestants are restricted to offering only contingent offers to foot-soldiers (and such offers are credible due to reputation effects). [Such a scenario might arise when the contestants are severely budget constrained before winning the prize. Alternatively, the underlying politico-legal scenario might be such that it becomes possible to effectively compensate one's own foot-soldiers only after having won the contest.] In this case, where $c^+ > 0$ while $k^+ = 0$, each contestant i will simultaneously announce c_i to maximize:

$$EW_i(c_i \mid c_j) \equiv P_i^*(c_i \mid c_j) \times [V - X(c_i, S_i^*(c_i \mid c_j))],$$

$$\text{where } P_i^*(c_i \mid c_j) = p_i(\theta) + \{0.5\phi.\delta.[p_i(\theta).c_i - p_j(\theta).c_j] / [\alpha - 0.5\phi.\delta.(c_i + c_j)]\},$$

$$\text{and } S_i^*(c_i \mid c_j) = 0.5\{1 + \delta.[p_i(\theta).c_i - p_j(\theta).c_j] / [\alpha - 0.5\phi.\delta.(c_i + c_j)]\}.$$ [21]

As in Section 3, we begin by imposing relatively mild restrictions on contingent cost function and present an equilibrium characterization result for the case of *only contingent compensations*.

Parameter restriction [X]: $X(c, S)$ is strictly increasing in its first argument and (weakly) increasing in its second argument, it is (weakly) convex in each of its arguments, with $X(0, S) = 0$, $[X_{cc}(c, S) + X_{SS}(c, S)] > 0$, and $X_{cS}(c, S) \geq 0$ for all $(c, S) \in [0, c^+] \times [0, 1]$.

PROPOSITION 4. Consider the case where $\{c^+ > 0, k^+ = 0\}$ and parameter restriction [X] holds. Then the contest is a *game with increasing best responses* that has at least one equilibrium. If $\{c_L^*, c_R^*\}$ is an equilibrium, then it is *bias-preserving* and satisfies the following property: for $i, j = L, R$, and $i \neq j$, $c_j^* \geq$ (resp., =) c_i^* whenever $p_i(\theta) >$ (resp., =) $p_j(\theta)$. If multiple equilibria exist, then the equilibria are 'ordered' in the following way: $\{c_L^{min}, c_R^{min}\}$, $\{c_L^1, c_R^1\}$, $\{c_L^2, c_R^2\}$, ... $\{c_L^{max}, c_R^{max}\}$, with $c_i^{min} < c_i^1 < c_i^2 < ... < c_i^{max}$ for $i = L, R$. $\{c_L^{min}, c_R^{min}\}$ is the *payoff-dominant* equilibrium; it is

[21] Here the contest success function $P_i^*(.)$ and the contest cost function $[P_i^*(.) \times X_i(.)]$ are "non-standard" in specific ways that make the contest a game of *increasing best-responses*.

the *null equilibrium* if $\{0.5\phi.V \leq \alpha.X_c(0, 0.5)\}$, and is a *strictly positive equilibrium* otherwise.[22] The following similarities and dissimilarities between Propositions 2 and 4 are to be noted. Firstly, while the contestants' best response functions are strictly increasing in both the contingent and the non-contingent offer games, there can be multiple Nash equilibria in the former game while the Nash equilibrium is dominance-solved in the latter game. Secondly, the larger is value of the prize and the effectiveness of foot-soldiering relative to the agents' allegiance value, the more likely is it that a contest equilibrium will be *positive* in both the contingent and the non-contingent offer games. Thirdly, in both games, an *ex ante underdog* cannot become an *ex post favourite* in any equilibrium (i.e., no equilibrium can be *bias-reversing*). Finally, while both candidates make identical offers when competing in the non-contingent compensations game, the *underdog* can make a strictly larger offer in a contingent-compensation equilibrium. As a result, it can be the case in the contingent-offers game that the contest equilibrium is *dissipative* for the *ex ante underdog* and not for the *ex ante favourite* (see Corollary 6 below).

Next, restricting ourselves to the scenario where the contestants can make only contingent offers, we now distinguish between two cases depending upon whether contestants compensate foot-soldiers *via* private goods (like cash) or *via* excludable public goods (like political access). Note that if a contestant's foot-soldiers are compensated *via* an excludable public good then the *army size* will not affect her total costs. In contrast, if compensation takes the form of paying in private goods, then it is likely that the product of 'per foot-soldier payoff' and 'own army size' will matter for each contestant. Given that, we present two alternative simplifications of the contest cost function $X(c, S)$ as follows, both of which satisfy parameter restriction [X].

Private-goods specification [XB]: $X(c, S) = B(c \times S)$ for all $(c, S) \in [0, c^+] \times [0, 1]$, where $B(.)$ satisfies: $B(0) = 0$, $B'(.) > 0$, and $B''(.) \geq 0$.

Public-good specification [XG]: $X(c, S) = G(c)$ for all $(c, S) \in [0, c^+] \times [0, 1]$, where $G(.)$ satisfies: $G(0) = 0$, $G'(.) > 0$, and $G''(.) > 0$.

PROPOSITION 5. Consider the case where $\{c^+ > 0, k^+ = 0\}$. If the cost function $X(c, S)$ satisfies the private goods specification [XB], then there exists a dominance-solved equilibrium $\{c_L^*, c_R^*\}$. If $\{\phi.\delta.V \leq \alpha.B'(0)\}$ then $c_L^* = c_R^* = 0$ for all $\theta \in [-½, ½]$; if $\{\phi.\delta.V > \alpha.B'(0)\}$ and $\theta = 0$ then $c_L^* = c_R^* \in (0, c^+]$; and if $\{\phi.\delta.V > \alpha B'(0)\}$ and $\theta \neq 0$ then either $\{c_U^* > c_F^* > 0\}$ or $\{c_U^* = c_F^* = c^+\}$.[23] Alternatively, if $X(c, S)$ satisfies the public good specification [XG], then there exists at least one equilibrium and every equilibrium is symmetric: $c_L^* = c_R^* = c^*$. If $\{\phi.\delta.V \leq \alpha.G'(0)\}$ then $c^* = 0$ in the payoff-dominant equilibrium. If $\{\phi.\delta.V > \alpha.B'(0)\}$ then every equilibrium is *positive*.

[22] In specific cases, the payoff-dominant equilibrium can also be the risk-dominant equilibrium.

[23] Recall that contestant F (respectively, U) refers to the *ex ante favourite* (respectively, *underdog*).

When $\theta \neq 0$ and $X(c, S)$ satisfies *either* [*XG*] *or* [*XB*], every *non-null equilibrium* is *bias-enhancing* and *dissipative for the underdog*, with $S_F^* > ½ > S_U^*$.

Proposition 5 establishes that for our 'contest model with only contingent offers', when the contingent contest cost function $X(c, S)$ satisfies either the private-goods specification [*XB*] or the public good specification [*XG*], the following properties hold in any *positive* equilibrium given an *ex ante favourite* and an *ex ante underdog*:

> The *underdog* offers contingent compensation that is no less than the *favourite*'s offer. In spite of that, the *underdog* acquires an *army of foot-soldiers* that is strictly smaller than that of the *favourite*. As a result, the *favourite* manages to extend her lead in the contest while the *underdog* falls farther behind.[24] Consequently, the *ex ante underdog* – one who tries at least as hard as her rival in recruiting foot-soldiers – is the contestant who will unambiguously gain from an externally-imposed ban on the recruitment of foot-soldiers.

An important difference between the private-goods specification and the public-good specification is the fact that the *underdog*'s contingent compensation offer can be strictly larger than that of the *favourite* in the former case, while the offers are necessarily equal in the latter case. This dichotomy is to be understood as follows. Starting from the situation where both contestants make identical offers, the *incremental cost* of raising the compensation amount for the *ex ante underdog* is smaller relative to that of the *favourite* under the private-goods specification than under the public-good specification. This is because when both contestants offer similar contingent compensation amounts, a larger measure of foot-soldiers join the *favourite*, thus raising her total (contingent) wage bill; and it is the wage bill $[c_i \times S_i]$ (rather than the offered compensation c_i) that determines the 'cost of keeping one's promise' under the private-goods specification. Thus, relative to the *favourite*, the *underdog* has a greater incentive to raise her offer under the private-goods specification than under the public-good specification.

We conclude our discussion of the 'contest with only contingent offer' model by emphasizing that in equilibrium, the *ex ante favourite* might indeed *gain* from being able to recruit foot-soldiers. Our next result presents a sufficient condition for this to happen in the scenario where contingent compensations are paid *via* an excludable public good.

COROLLARY 6. Consider the case where $\{c^+ > 0, k^+ = 0\}$ and the cost function satisfies the public good specification [*XG*] with $G(c) = c^2$. Then there exist parameter values $\{V, \alpha, \delta, \phi, c^+\}$ and

[24] In this regard, the *underdog* might turn out to be 'doubly disadvantaged' as compared to the *favourite* in the following sense: The equilibrium total contest cost of the *underdog* may be greater than that of the *favourite*, while her equilibrium probability of winning the contest will be less than (not only the *favourite*'s equilibrium probability of winning but) her own initial probability of winning.

$\{|\theta| > ⅓\}$ such that: there exists $c^* \in (0, c^+)$ for which $c_L^* = c_R^* = c^*$ in the unique equilibrium; and in equilibrium, the *ex ante favourite* strictly benefits from being able to recruit foot-soldiers.

5. Contest Equilibria with Contingent & Non-contingent Compensations

We now turn our attention to the scenario where contestants can offer contingent as well as non-contingent compensations to the agents (i.e., $c^+ > 0$ and $k^+ > 0$). In this general scenario, each contestant i's payoff function might not be quasi-concave jointly in (c_i, k_i), and as a result, the existence of a pure-strategy Nash equilibrium is not guaranteed. In what follows, we study two special scenarios with the following features. In both scenarios we posit that the non-contingent compensation is paid in cash. In contrast, the contingent compensation is paid in cash in one scenarios, and is paid *via* an excludable public good in the other scenarios. Specifically, the two scenarios that we study are the following:

The private-private scenario: $\{c^+ > 0, k^+ > 0\}$ with $X(c, S) = \beta.c.S$ for all $(c, S) \in [0, c^+] \times [0, 1]$; and $Y(k, S) = \gamma.k.S$ for all $(k, S) \in [0, k^+] \times [0, 1]$ with $min\{(\beta/\delta), \gamma\} \geq 1$.

The private-public scenario: $\{c^+ > 0, k^+ > 0\}$ with $X(c, S) = G(c)$ for all $(c, S) \in [0, c^+] \times [0, 1]$ with $G(0) = 0$, $G'(.) > 0$, and $G''(.) > 0$; and $Y(k, S) = \gamma.k.S$ for all $(k, S) \in [0, k^+] \times [0, 1]$.

PROPOSITION 7. Consider the private-private scenario. An equilibrium exists in this scenario. If there are multiple equilibria, they are *ordered* and the *minimal* equilibrium is payoff-dominant.

[A] In the case where $(\beta/\delta) < \gamma$, all equilibria have the following features. If $\theta = 0$ then $c_L^* = c_R^*$ and $k_L^* = k_R^*$, while if $\theta \neq 0$ then $c_U^* \geq c_F^*$ and $k_U^* \geq k_F^*$. If $c_i^* < c^+$ then $k_i^* = 0$, and if $k_i^* > 0$ then $c_i^* = c^+$. A *non-null* equilibrium is *bias-enhancing* and *underdog-dissipative* with $S_F^* > ½ > S_U^*$.

[B] In the case where $(\beta/\delta) > \gamma$, all equilibria have the following features. If $\theta = 0$ then $c_L^* = c_R^*$ and $k_L^* = k_R^*$, while if $\theta \neq 0$ then $c_U^* \geq c_F^*$ and $k_U^* = k_F^*$. If $k_i^* < k^+$ then $c_i^* = 0$, and if $c_i^* > 0$ then $k_i^* = k^+$. A *non-null* equilibrium that contains no contingent compensation is *bias-maintaining* and *dissipative for both contestants* with $S_F^* = ½ = S_U^*$; in contrast, a *non-null* equilibrium containing contingent compensation is *bias-enhancing* and *dissipative for the underdog*, with $S_F^* > ½ > S_U^*$.

Proposition 7 splits the private-private scenario into two cases: In case [A] – with $(\beta/\delta) < \gamma$ – it is cheaper at the margin to compensate foot-soldiers *via* contingent offers as compared to non-contingent offers. In this case, both contestants exhaust all feasible contingent payments before utilizing non-contingent payments in any positive equilibrium. The use of contingent payments causes the equilibrium to be *bias-enhancing* and *underdog-dissipative*. In case [B] – with $(\beta/\delta) > \gamma$ – it is cheaper at the margin to compensate foot-soldiers *via* non-contingent offers. So,

in any positive equilibrium both contestants exhaust non- contingent compensation opportunities before utilizing contingent payments. In this case, only if the contestants use contingent compensations is the equilibrium is *bias-enhancing* and *underdog-dissipative* .

Proposition 7 indicates that small changes in the foot-soldiers' discount factor δ and/or the contestants' 'costs of capital parameters' $\{\beta, \gamma\}$ can discontinuously change the *nature* of foot-soldier cash compensations (from non-contingent to contingent compensations). Furthermore, the *magnitudes* of these cash offers can also change discontinuously, and in ways that make both contestants strictly better off in the 'non-contingent compensation equilibrium' as compared to the 'contingent compensation equilibrium'. Specifically, we can construct examples exhibiting the following phenomenon: We start from the case where $(\beta/\delta) = \gamma + \eta$ (where η positive, small), and each contestant i offers $\{k_i^*=k^* > 0,\ c_i^*=0\}$ in equilibrium. Then the foot-soldiers become a 'bit more patient' (δ rises incrementally) to force $(\beta/\delta) = \gamma - \eta$, so that in the *new* equilibrium each contestant i offers $\{k_i^*=0,\ c_i^*=c^* > 0\}$. Here, c^* can be significantly larger than k^*, so much so that each contestant i's expected payoff in the 'contingent compensations equilibrium' $EW_i(c^*, c^*)$ can be lower than her expected payoff in the 'non-contingent compensations equilibrium' $EW_i(k^*, k^*)$ (even though c^* is paid only when the contestant wins).

PROPOSITION 8. Consider the private-public scenario with $[G'(0)/\delta] \geq \gamma \geq 1$. In this case, an equilibrium exists, and all equilibria are *symmetric*. If $\phi.V \leq \alpha.\gamma$, then the unique equilibrium is *null* with $(c_L^*=c_R^*=0, k_L^*=k_R^*=0)$. If $\phi.V \in (\alpha.\gamma, \{\gamma.k^+ + 2\alpha.G'(0)/\delta\}]$, then every equilibrium is *non-null*, with the payoff-dominant equilibrium having $(c_L^*=c_R^*=0, k_L^*=k_R^*>0)$ and being *bias-maintaining*. If $\phi.V > [\gamma k^+ + 2\alpha G'(0)/\delta]$, then every equilibrium is *positive* with $(c_L^*=c_R^*>0, k_L^*=k_R^*=k^+)$ and is *bias-enhancing* and *dissipative for the underdog*, with $S_F^* > ½ > S_U^*$.

Proposition 8 focuses on the case where all contingent-compensation needs to be paid *via* access to an excludable public goods (while the non-contingent payment is made in cash), and where the incremental cost of initiating contingent compensations is not negligible (with $[G'(0)/\delta] \geq \gamma$). In this case, it is cheaper for each contestant to first exhaust all non- contingent compensation opportunities before starting contingent payments.

We conclude this section by emphasizing that we consider the hypotheses of Proposition 7 to be capturing a set of conditions that might be quite prevalent in the real-world: Contestants cannot compensate their foot-soldiers *via* access to certain kinds of excludable public goods *before* winning the contest, while they find it much easier to do so *after* they have won. At the same time, various legal compulsions limit the extent of up-front (non-contingent) cash compensations that they can pay to each of their foot-soldiers. As a result, they offer limited (and equal) non-

contingent cash compensations and supplement them by additional (and equal) contingent compensations *via* some excludable public good. Note that such an equilibrium is necessarily characterized by the following features: The *ex ante favourite* extends hear lead in the contest by attracting a larger foot-soldier army, and the *ex ante underdog* is unambiguously worse-off than she would be if foot-soldier recruitment was disallowed by law.

6. A Two-stage Contest

Our formal model of a 'contest with foot-soldiers' begins with the realization of $\theta \in [-\frac{1}{2}, \frac{1}{2}]$, which determines the *initial win probabilities* of the two contestants. In this regard, it is quite plausible to take the view that the realized θ depends upon some inherent attributes and/or prior investments of the contestants. To the extent that the realization of θ is at least partially dependent on the contestants' *prior choices*, it will be natural to enquire as to how the possibility of a future 'contest with foot-soldiers' affects these choices (especially when these choices might have significant welfare consequences for the rest of the economy). In what follows, we briefly address this question within the context of a particular version of our contest model.

Recall our *Story* 1: Two firms – L and R – contest to secure a public project in a locality. The firms first submit 'technical bids', and then an 'expert committee' deliberates and awards the contract to one firm. Let us posit that when a particular firm gets the contract, the government compensates the firm for all costs incurred to complete the project and then gives the firm a 'remuneration' of V.

In this scenario, we think of θ being generated in the following manner: Each firm i has a commonly-known inherent skill level $h_i \in (0, 1)$, and then chooses a level of investment $q_i \geq 0$ in 'knowledge acquisition' at a (strictly convex) cost $Q(q_i)$. This generates the 'expertise bias parameter' $\theta(h_L, q_L; h_R, q_R) \in [-\frac{1}{2}, \frac{1}{2}]$, which is strictly increasing in its first two arguments and strictly decreasing in its last two arguments.

After $\theta(.)$ is realized, the two firms play the 'only contingent offers' contest-game as described in Section 4, with the contingent cost function satisfying the *public good specification*. Specifically, each firm i offers a contingent compensation amount c_i to local foot-soldiers (to lobby before the expert committee before it starts its deliberations), and at this point in time, the aim of each firms i is to maximize: $EW_i(c_i, c_j \mid \theta(.)) \equiv P_i^*(c_i \mid c_j) \times [V - (c_i)^n]$, given $n > 1.5$ and $P_i^*(c_i \mid c_j)$ and $S_i^*(c_i \mid c_j)$ as defined in Section 4.

It can be proved that in this case there exists $c^* \in (0, c^+]$ such that in the unique equilibrium of the second-stage contest, the firms make contingent compensation offers: $c_L^* = c_R^* = c^*$. Given

that contest equilibrium, the firms' first-stage interaction simplifies as follows: Given $\{h_L, h_R\}$, firm L chooses $q_L \geq 0$ to maximize: $\{[\alpha(1+\theta(.)) - \phi.\delta.c^*] / [2\alpha - \phi.\delta.c^*]\}.[V-(c^*)^n] - Q(q_L)$ and firm R chooses $q_R \geq 0$ to maximize: $\{[\alpha(1-\theta(.)) - \phi.\delta.c^*] / [2\alpha - \phi.\delta.c^*]\}.[V-(c^*)^n] - Q(q_R)$. Let $\{q_L^*, q_R^*\}$ be the unique interior equilibrium in this first-stage interaction.

Recognize that if recruiting foot-soldiers was not an option, then given $\{h_L, h_R\}$, firm L would choose $q_L \geq 0$ to maximize $\{[0.5(1+\theta(.))].V - Q(q_L)\}$, while firm R would choose $q_R \geq 0$ to maximize $\{[0.5(1-\theta(.))].V - Q(q_R)\}$. Let $\{q_L^0, q_R^0\}$ be the unique interior equilibrium in this case. It is then easy to establish that for each contestant i, $q_i^* < q_i^0$ if and only if $\alpha.(c^*)^{n-1} > \phi.\delta.V$.

Thus, under plausible conditions it can be the case that the possibility of contesting *via* the use of foot-soldiers depresses socially beneficial *ex ante* investments that the firms would have otherwise made. This result highlights a possible negative social impact of the presence of unemployed / underemployed youth who can be used as foot-soldiers in contests.

7. Robustness Checks

Our analysis of *contests with foot-soldiers* has been carried out under two maintained assumptions. Firstly, we have assumed that the two contestants are identical in every respect except for their initial win probabilities. Secondly, we have assumed that the difference in the sizes of the contestants' foot-soldier armies alter their chances of winning in a linear manner. In this concluding section, we briefly comment on the robustness of our results with respect to these assumptions.

Our main conclusions have been that contesting with foot-soldiers *preserves* (and in some cases *enhances*) the initial bias in win probabilities, and that such contesting is necessarily *dissipative* for the *ex ante underdog*. How will these conclusions change if the value of the prize (V) is not the same for the two contestants? Note that the major change will be the following: When $V_i > V_j$ for some $i, j = L, R$, and $i \neq j$, then (for a set of parameter configurations), contestant i will offer greater (contingent *and* non-contingent) compensations than contestant j (irrespective of whether i is the *ex ante favourite* or not). As a result, more foot-soldiers will join i's army, and that will raise i's final win probability. So, if i was indeed the *ex ante underdog*, equilibrium-contesting by the two players might indeed lead to *bias-reversion*, but only because the prize is significantly more attractive to the *ex ante underdog* than to the *ex ante favourite*; further, as contestant i will have to incur a greater cost to improve her chances of winning, it is not guaranteed that the contest will not be *dissipative* for her. Note that basic continuity arguments can be used in our model to show that as long as $|V_L - V_R|$ is sufficiently small, our qualitative results regarding the contest equilibria will continue to hold.

Next, we focus on the following question: How critical to our results is the *difference-form* 'impact of foot-soldiering' specification: $P(S_i, S_j, p_i) = [p_i + 0.5\phi.(S_i - S_j)]$? In this respect, we have the following result:

PROPOSITION 9. Suppose $P(S_i, S_j, p_i)$ satisfies the following properties on its domain: $P_3(.) > 0$, $0 < P_1(.) < ¼$, $-¼ < P_2(.) < 0$; $P(S_i, S_j, p_i) + P(S_j, S_i, 1-p_i) = 1$; and $[P(S_i, S_j, p_i) - p_i]$ has the same sign as $[S_i - S_j]$. Consider the case where $\{c^+ > 0, k^+ = 0\}$ and $\theta \neq 0$.

[A] If the contingent cost function $X(c, S)$ satisfies the private goods specification [*XB*], then every pure-strategy equilibrium (if one exists) will have $c_U^* \geq c_F^* \geq 0$, and every *non-null* equilibrium (if one exists) will be *bias-enhancing*, and so will be *underdog-dissipative*.

[B] If $X(c, S)$ satisfies the public-good specification [*XG*], then every pure-strategy equilibrium (if one exists) will be symmetric, with $c_U^* = c_F^* \geq 0$; and every *non-null* equilibrium (if one exists) will be *bias-enhancing*, and so will be *underdog-dissipative*.

The above result clarify that: (*a*) when the contestants can only offer contingent compensations, the different properties of contest equilibria that we uncover are quite robust to alternative specifications of the $P(S_i, S_j, p_i)$ function, but (*b*) we might not be able to guarantee the existence of pure-strategy Nash equilibria for such general functional forms.[25] The extension of Proposition 9 to the case where contestants can offer both contingent and non-contingent compensations awaits further research.

[25] Existence of equilibrium is guaranteed for the following linear-quadratic' specification: $P(S_i, S_j, p_i) = p_i + 0.5[\Phi(S_i) - \Phi(S_j)]$ for $i, j = L, R$, and $i \neq j$, where $\Phi(S) = \phi_1.S + \phi_2.S^2$ with $\phi_1 \geq 0$, $[\phi_1 + 2\phi_2] > 0$ and $[\phi_1 + \phi_2] < 0.5$.

Appendix: Story of a Contest for Public Project with Lobbying by Foot-soldiers

Recall our *Story* 1: Two firms – L and R – contest to secure a public project in a locality. The firms first submit 'technical bids', and then an 'expert committee of bureaucrats' deliberates and awards the contract to one firm. When a particular firm gets the contract, the government compensates the firm for all costs incurred for the project and then gives the firm remuneration V.

Let us first consider the scenario where no firm can hire local people to 'lobby for the firm' in front of the expert committee before it begins its deliberations. Consider the following question: "Given the public bids of the two firms, how do *outsiders* (i.e., the firms and all economic agents who are not members of the expert committee) assign the chance of a particular firm winning the project?" We formulate the answer to this question as follows:

The publicly observed bids generate a 'technological bias value' $\theta \in [-\frac{1}{2}, \frac{1}{2}]$. While the committee gives some weight to this bias, it is also driven by (hidden) political compulsions that force *outsiders* to treat its decision-making process as stochastic. Specifically, the *outsiders* posit that the committee's decision is generated *via* the following model. Define the variable $\Delta = \theta + \omega$ where ω is a random variable that is uniformly distributed on the support $[-1, 1]$, and posit that given θ, the committee (in its secret deliberations) will award the project to firm L if and only if the 'realized Δ given θ' is positive (i.e., the realized ω is greater than $[-\theta]$). Then following this model, the *outsiders*' probability belief (at the time that the committee begins deliberations) that firm L will win the contract is: $p_L(\theta) = 0.5(1+\theta)$.

Next consider the case where, between the time that the firms submit their technical bids (thus generating θ) and the committee begins its deliberations, each firm can recruit local foot-soldiers who can go and lobby before the committee to award the contract to their employer firms. The *outsiders* posit that the committee's post-lobbying decision is generated *via* the same $[\Delta = \theta + \omega]$ model as before, except that now the random variable ω is drawn from a uniform distribution on the support $[-1+\phi.(S_L - S_R),\ 1+\phi.(S_L - S_R)]$. Consequently, the *outsiders*' probability belief (at the time that the committee goes into post-lobbying deliberations) that firm L will win the contract is: $P_L(\theta) = 0.5[1+\theta+\phi.(S_L - S_R)]$. Our example thus provides a simple micro-foundation for interpreting $\{p_i(\theta), P_i(\theta)\}$ as the *initial* and *final win probabilities* of contestants i, for $i = L, R$.

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