Supplementary Material for

"Appropriative Conflicts and the Evolution of Property Rights"

Proofs not for publication

PROOFS OF COROLLARY 3 AND PROPOSITION 4:

We first determine how $W_2^\dagger(.)$, $[W_1^\dagger(.) + W_2^\dagger(.)]$, $W_1^\dagger(.)$, and $[W_1^\dagger(.) - W_2^\dagger(.)]$ change in π. To simplify notation, we set $R = 1$ as R does not affect these comparative statics results.

We establish some preliminary facts. First, $W_1^\dagger(.)$ and $W_2^\dagger(.)$ are continuous in all their arguments. Next, given any $\pi < 1$, the Banditry equilibrium exists for all $\rho \leq \tilde{\rho}(\pi \mid \sigma)$ while the Hobbesian equilibrium exists for all $\rho > \tilde{\rho}(\pi \mid \sigma)$. Further, $\tilde{\rho}(\pi \mid \sigma)$ strictly decreases in π and takes its maximum value of $0.5\sigma/(1+\sigma) < 0.25$ at $\pi = 0$. Thus, for $\rho > 0.5\sigma/(1+\sigma)$, we obtain the Hobbesian equilibrium for all $\pi \in [0, 1)$. Rearranging $\tilde{\rho}(\pi \mid \sigma) \equiv 0.5\{[(1-\pi)\sigma]/[1+\pi+(1-\pi)\sigma]\}$, we obtain the following result: for any $\rho \in (0, 0.5\sigma/(1+\sigma)]$, the Banditry equilibrium exists for all $\pi \in [0, \pi^0(\rho \mid \sigma)]$, while the Hobbesian equilibrium exists for all $\pi \in (\pi^0(\rho \mid \sigma), 1)$, where $\pi^0(\rho \mid \sigma) \equiv [\sigma - 2\rho(1 + \sigma)]/[\sigma + 2\rho(1 - \sigma)] \in [0, 1)$.

Consider the case where $\rho > 0.5\sigma/(1+\sigma)$ and thus the Hobbesian equilibrium obtains for all $\pi \in [0, 1)$. In this case, $\partial W_2^\dagger(.)/\partial\pi = \sigma/[(1 + \sigma) + (1 - \sigma)\pi]^2 - ½ + \rho$. As this expression is strictly falling in π, $W_2^\dagger(.)$ is strictly concave in π. So, there are the following three mutually exclusive cases to consider:

- $\rho \in [½ - \sigma/4, ½) \Leftrightarrow \partial W_2^\dagger(\sigma, \pi = 1, R, \rho)/\partial\pi \geq 0$: in this case $W_2^\dagger(\sigma, \pi, R, \rho)$ is uniquely maximized at $\pi = 1$. [Note that there is no discontinuity in $W_2^\dagger(\sigma, \pi, R, \rho)$ at $\pi = 1$.]
- $\rho \in (0.5\sigma/(1+\sigma), ½ - \sigma/(1 + \sigma)^2] \Leftrightarrow \partial W_2^\dagger(\sigma, \pi = 0, R, \rho)/\partial\pi \leq 0$: in this case $W_2^\dagger(\sigma, \pi, R, \rho)$ is uniquely maximized at $\pi = 0$.
- $\rho \in (½ - \sigma/(1 + \sigma)^2, ½ - \sigma/4) \Leftrightarrow \partial W_2^\dagger(\sigma, \pi = 1, R, \rho)/\partial\pi < 0 < \partial W_2^\dagger(\sigma, \pi = 0, R, \rho)/\partial\pi \Leftrightarrow$: in this case $W_2^\dagger(\sigma, \pi, R, \rho)$ is uniquely maximized at $\pi = \tilde{\pi}(\rho \mid \sigma)$ (since $\partial W_2^\dagger(.)/\partial\pi = 0$ at $\pi = \tilde{\pi}(\rho \mid \sigma)$).

Next, we consider the case when $\rho \leq 0.5\sigma/(1+\sigma)$. In this case, we obtain the Hobbesian equilibrium for all $\pi \in (\pi^0(\rho \mid \sigma), 1)$. In this sub-case, the Hobbesian equilibrium expression for $W_2^\dagger(.)$ is strictly concave in π and $\partial W_2^\dagger(\sigma, \pi = 0, R, \rho)/\partial\pi < 0$; thus $W_2^\dagger(.)$ is strictly decreasing in $\pi \in (\pi^0(\rho \mid \sigma), 1)$.[1] We obtain the Banditry equilibrium for the case when $\{\rho \in (0, 0.5\sigma/(1+\sigma)]$ and $\pi \in [0, \pi^0(\rho \mid \sigma)]\}$.

[1] Note that the Hobbesian equilibrium does not obtain here for $\pi \leq \pi^0(\rho \mid \sigma)$, and we only have: $W_2^\dagger(\sigma, \pi, R, \rho) = \left\{\pi\rho + \frac{[1-\pi][1+(1-\sigma)\pi]}{2[1+\sigma+(1-\sigma)\pi]}\right\}.R$ for $\pi \in (\pi^0(\rho \mid \sigma), 1)$. But the function $\left\{\pi\rho + \frac{[1-\pi][1+(1-\sigma)\pi]}{2[1+\sigma+(1-\sigma)\pi]}\right\}.R$ is well-defined for all $\pi \in [0, 1]$. So, if we show that this function is strictly decreasing for $\pi \in [0, 1]$, then we would have also shown that $W_2^\dagger(\sigma, \pi, R, \rho)$ is strictly decreasing for $\pi \in (\pi^0(\rho \mid \sigma), 1)$.

We next show that for these parameter values: $\partial W_2^\dagger(.)/\partial\pi < 0$. Using the Banditry equilibrium expression for $W_2^\dagger(.)$, we get $\partial W_2^\dagger(.)/\partial\pi = -(1-\sigma)/2 + \sigma\rho + \{[1+\pi + (1-\pi)\sigma]^{1/2}.[(1-\sigma).[2(1-\pi)\sigma\rho]^{1/2} - [\pi + (1-\pi)\sigma].(\sigma\rho) / [2(1-\pi)\sigma\rho]^{1/2}] - [\pi + (1-\pi)\sigma].[2(1-\pi)\sigma\rho]^{1/2}(1-\sigma) / [2(1+\pi + (1-\pi)\sigma)^{1/2}]\} / \{1+\pi + (1-\pi)\sigma\} = -(1-\sigma)/2 + \sigma\rho + (\sigma\rho).\{[\pi + (1-\pi)\sigma].[2(1-\sigma).(1-\pi)-1] + [3(1-\sigma).(1-\pi)-1]\} / \{[1+\pi + (1-\pi)\sigma]^{3/2}.[2(1-\pi)\sigma\rho]^{1/2}\}$. Evaluating the previous expression at $\pi = 0$, we get: $\partial W_2^\dagger(\pi{=}0)/\partial\pi = -(1-\sigma)/2 + \sigma\rho - [2\sigma\rho]^{1/2}.[\sigma^2 + \sigma - 1] / [1+\sigma]^{3/2}$, which is easily shown to be strictly negative for all $\sigma \in [0.5, 1)$ and $\rho \in (0, 0.5\sigma/(1+\sigma)]$. Finally, straightforward differentiation followed by some algebraic re-arrangement helps us sign $\partial^2 W_2^\dagger(.)/\partial\pi^2$ as negative, thus giving us the result that $\partial W_2^\dagger(.)/\partial\pi < 0$ for the case when $\{\rho \in (0, 0.5\sigma/(1+\sigma)]$ and $\pi \in [0, \pi^0(\rho \mid \sigma)]\}$.

We thus conclude that when $\rho \in (0, \rho^-(\sigma)]$, where $\rho^-(\sigma) \equiv ½ - \sigma/(1 + \sigma)^2)$, $W_2^\dagger(.)$ strictly decreases in π and is maximized at $\pi = 0$.

We now show that $[W_1^\dagger(.) + W_2^\dagger(.)]$ strictly increases in $\pi \in [0,1)$. For the case when $\{\rho > \tilde{\rho}(\pi|\sigma)\}$ or $\{\rho \leq \tilde{\rho}(\pi|\sigma)$ and $\pi > \pi^0(\rho \mid \sigma)\}$ (i.e., when a Hobbesian equilibrium exists): $[W_1^\dagger(.) + W_2^\dagger(.)] = \{\pi + (1-\pi)[1 + (1-\sigma)\pi]/[1+\sigma+(1-\sigma)\pi]\} \Rightarrow \partial(W_1^\dagger(.) + W_2^\dagger(.))/\partial\pi = 2\sigma/[1+\sigma+(1-\sigma)\pi]^2 > 0$. For the case when $\{\rho \leq \tilde{\rho}(\pi|\sigma)$ and $\pi \leq \pi^0(\rho \mid \sigma)\}$ (i.e., when a Banditry equilibrium exists): $[W_1^\dagger(.) + W_2^\dagger(.)] = 1 - \sqrt{\{2(1-\pi)\sigma\rho / [1+\pi+(1-\pi)\sigma]\}}$. Note that the denominator of the second term of the previous expression increases in π while the numerator decreases in π, so the whole expression is easily seen to be increasing in π. So, in this case also we have that $\partial[W_1^\dagger(.) + W_2^\dagger(.)]/\partial\pi > 0$. Hence, $[W_1^\dagger(.) + W_2^\dagger(.)]$ strictly increases in $\pi \in [0,1]$, and is maximized at $\pi = 1$.

We next establish that $W_1^\dagger(.)$ strictly increases in π. It is readily established that for the case when $\{\rho > \tilde{\rho}(\pi|\sigma)\}$ or $\{\rho \leq \tilde{\rho}(\pi|\sigma)$ and $\pi > \pi^0(\rho \mid \sigma)\}$: $\partial W_1^\dagger(.)/\partial\pi = \sigma/[(1 + \sigma) + (1 - \sigma)\pi]^2 - ½ + (1 - \rho) > 0$. Further, since $W_2^\dagger(.)$ strictly decreases in π while $[W_1^\dagger(.) + W_2^\dagger(.)]$ strictly increases in π for $\{\rho \leq \tilde{\rho}(\pi|\sigma)$ and $\pi \leq \pi^0(\rho \mid \sigma)\}$, it must be that in this case $W_1^\dagger(.)$ strictly increases in π.

We finally establish that $[W_1^\dagger(.) - W_2^\dagger(.)]$ strictly increases in π. When $\{\rho > \tilde{\rho}(\pi|\sigma)\}$ or $\{\rho \leq \tilde{\rho}(\pi|\sigma)$ and $\pi > \pi^0(\rho \mid \sigma)\}$, $[W_1^\dagger(.) - W_2^\dagger(.)] = \pi.(1-2\rho) \Rightarrow \partial(W_1^\dagger(.) - W_2^\dagger(.))/\partial\pi > 0$. When $\{\rho \leq \tilde{\rho}(\pi|\sigma)$ and $\pi \leq \pi^0(\rho \mid \sigma)\}$, $W_2^\dagger(.)$ strictly decreases in π and $W_1^\dagger(.)$ strictly increases in π, implying that $[W_1^\dagger(.) - W_2^\dagger(.)]$ strictly increases in π.

Thus, we have proved that $\partial W_1^\dagger(.)/\partial\pi > max\{0, \partial W_2^\dagger(.)/\partial\pi\}$. Henceforth, we allow any $R > 0$.

Note that $[W_2^\dagger(.)/R_2] = [W_1^\dagger(.)/R_1] = 1$ when $\pi = 1$. Thus, when $\rho \in (0, \rho^-(\sigma)]$, $[W_2^\dagger(.)/R_2] > 1 > [W_1^\dagger(.)/R_1]$ for all $\pi < 1$ since in this case, $W_2^\dagger(.)$ increases as π falls while $W_1^\dagger(.)$ decreases as π falls.

Next, we prove that $[W_2^\dagger(.)/R_2] > [W_1^\dagger(.)/R_1]$ for all $\pi < 1$ and $\rho \in (\rho^-(\sigma), ½)$. In this case, where the Hobbesian equilibrium obtains for all $\pi < 1$, let us suppose that $[W_2^\dagger(.)/R_2] \leq [W_1^\dagger(.)/R_1]$. Then, using the expressions for $W_1^\dagger(.)$ and $W_2^\dagger(.)$ in Proposition 2 and doing straightforward algebra gives us that $\rho \geq ½$, which is a contradiction to the fact that $\rho < ½$. Hence, our supposition is incorrect. We thus have $[W_2^{\dagger}(.)/R_2] > [W_1^{\dagger}(.)/R_1]$ for all $\pi < 1$ and $\rho \in (0, ½)$.

To establish that $b_{12}^{\dagger}(\sigma, \pi, R, \rho) > 0$ for all $\pi < 1$ and $\rho \in (0, ½)$, we first establish that $[B_2^{\dagger}(.)/R_2] < [B_1^{\dagger}(.)/R_1]$. This is true when the Banditry equilibrium obtains since then $B_2^{\dagger}(.) = 0$. Note that in the Hobbesian equilibrium we have that: $G_2^{\dagger}(.) = G_1^{\dagger}(.) \Rightarrow [G_2^{\dagger}(.)/R_2] > [G_1^{\dagger}(.)/R_1]$, hence: $[B_2^{\dagger}(.)/R_2] = [(R_2 - G_2^{\dagger}(.))/R_2] < [B_1^{\dagger}(.)/R_1] = [(R_1 - G_1^{\dagger}(.))/R_1]$. Hence, we have that: $[B_2^{\dagger}(.)/R_2] < [B_1^{\dagger}(.)/R_1]$ for all $\pi < 1$ and $\rho \in (0, ½)$. Now, by definition of $b_{12}^{\dagger}(\sigma, \pi, R, \rho)$, we have that: $W_2^{\dagger}(.) = [B_2^{\dagger}(.) + b_{12}^{\dagger}(\sigma, \pi, R, \rho)]$ and $W_1^{\dagger}(.) = [B_1^{\dagger}(.) - b_{12}^{\dagger}(\sigma, \pi, R, \rho)]$. So, we have: $\{W_2^{\dagger}(.)/R_2\} > \{W_1^{\dagger}(.)/R_1\} \Rightarrow \{[B_2^{\dagger}(.) + b_{12}^{\dagger}(\sigma, \pi, R, \rho)]/R_2\} > \{[B_1^{\dagger}(.) - b_{12}^{\dagger}(\sigma, \pi, R, \rho)]/R_1\} \Rightarrow b_{12}^{\dagger}(\sigma, \pi, R, \rho).[1/R_1 + 1/R_2] > [\{B_1^{\dagger}(.)/R_1\} - \{B_2^{\dagger}(.)/R_2\}] > 0$, hence we get that: $b_{12}^{\dagger}(\sigma, \pi, R, \rho) > 0$.

Finally, consider the rights negotiation game given $\Pi = [0, 1]$. Given the poorer community 2's optimal rights choice $\pi^{\dagger}(\rho \mid \sigma)$ for any $\rho \in (0, ½)$: if community 1 announces $s_1 \leq \pi^{\dagger}(\rho \mid \sigma)$, then it is optimal for community 2 to continue by announcing any $s_2 \in [s_1, 1]$ [since $W_2^{\dagger}(.)$ is single-peaked at $\pi = \pi^{\dagger}(\rho \mid \sigma)$] leading to the establishment of s_1 ($\leq \pi^{\dagger}(\rho \mid \sigma)$) as the rights regime; if community 1 announces $s_1 > \pi^{\dagger}(\rho \mid \sigma)$, then it is optimal for community 2 to continue by announcing $s_2 = \pi^{\dagger}(\rho \mid \sigma)$ leading to the establishment of $\pi^{\dagger}(\rho \mid \sigma)$ as the rights regime. Since $W_1^{\dagger}(.)$ is strictly increasing in π, in equilibrium, either community 1 will announce any $s_1 \in (\pi^{\dagger}(\rho \mid \sigma), 1]$ and community 2 will optimally continue by announcing $s_2 = \pi^{\dagger}(\rho \mid \sigma)$, or community 1 will announce $s_1 = \pi^{\dagger}(\rho \mid \sigma)$ and community 2 will optimally continue by announcing any $s_2 \in [\pi^{\dagger}(\rho \mid \sigma), 1]$, each case leading to the unique *SPNE* outcome of the establishment of $\pi^{\dagger}(\rho \mid \sigma)$ as the rights regime. □

PROOF OF PROPOSITION 5:

The detailed proof of the proposition is available in Goel (2017).[2] Here we simply record that in the guns-and-butter game with no commitment, the conclusions of Lemma 1 hold, and given $\pi \in [0, 1)$, the unique equilibrium payoffs of the two communities in the guns-and-butter game are:

$$W_1^{[N]}(\pi, \rho) = \begin{cases} \{\pi(1-\rho) + [(1-\pi)\sigma]/4\}[R_1 + R_2] & \text{for } \rho \in (\rho^N(\pi, \sigma), ½], \\ \{\sqrt{[\pi + (1-\pi)\sigma]} - \sqrt{[(1-\pi)\sigma\rho]}\}^2[R_1 + R_2] & \text{for } \rho \in (0, \rho^N(\pi, \sigma)], \end{cases}$$

$$W_2^{[N]}(\pi, \rho) = \begin{cases} \{\pi\rho + [(1-\pi)\sigma]/4\}[R_1 + R_2] & \text{for } \rho \in (\rho^N(\pi, \sigma), ½], \\ \{\sqrt{[[\pi + (1-\pi)\sigma][(1-\pi)\sigma\rho]]} - (1-\pi)\sigma\rho\}[R_1 + R_2] & \text{for } \rho \in (0, \rho^N(\pi, \sigma)]; \end{cases}$$

where $\rho^N(\pi, \sigma) \equiv 0.25\{[(1-\pi)\sigma]/[\pi + (1-\pi)\sigma]\} \in (0, 0.25]\ \forall \pi \in [0, 1)$. If $\pi = 1$, then $W_i^{[N]}(\pi, \rho) = R_i$. Then, differentiation of the payoff functions with respect to π (for ρ in $(\rho^N(\pi,\sigma), ½]$ and in (0,

[2] Goel, B. (2017) "Contest Theory and its Applications to Development and Political Economy," *under preparation*, Indian Institute of Management Calcutta.

$\rho^N(\pi, \sigma)]$) proves that $W_1^{[N]}(\pi, \rho)$ is maximized at 1 and $W_2^{[N]}(\pi, \rho)$ is maximized at $\pi^{[N]}(\rho \mid \sigma)$. □

PROOF OF PROPOSITION 6:

Let communities $i, j = 1, 2$ with $i \neq j$. Also, define $R \equiv (R_1 + R_2)$. We restrict attention to $\rho > 0.25$, hence: $0.75R > R_1 > R_2 > 0.25R$. It is easy to see that (the counterpart of) Lemma 1 and its proof go though without change with this alternative specification: Given $\pi \in [0, 1)$ and given a guns-and-butter *output vector* $\{(g_1, B_1), (g_2, B_2)\}$ such that $[B_1 + B_2] > 0$ and $[g_1 \times g_2] > 0$, community i's final payoff (after the exchange of the butter bribe) is:

$$\{[0.5(1-\pi)(1-\sigma)(B_1+B_2)] + [\pi B_i + (1-\pi)[g_i / (g_1+g_2)].\sigma.(B_1+B_2)]\}.^3$$

The proof of (the counterpart of) Proposition 2 requires minor adjustments, which are described next. If $\pi = 1$, then in the unique guns-and-butter equilibrium: $W_k^{\dagger} = B_k^{\dagger} = R_k$ for $k = 1, 2$. Given $\pi \in [0, 1)$ we want to determine all the guns-and-butter equilibria. It is straightforward to see that setting $[G_i + B_i] < R_i$ is a strictly dominated strategy. Henceforth we will take $[G_i + B_i] = R_i$ and consider G_i as community i's sole choice variable; further, we will denote an equilibrium simply by $\{G_1^{\dagger}, G_2^{\dagger}\}$. We will first establish that there exists a unique *candidate* 'interior equilibrium' $\{G_1^{\dagger}, G_2^{\dagger}\} \in (0, R_1)\times(0, R_2)$. Given a conjecture about rival $G_j \in (0, R_j)$ (resp., $G_j = R_j$), community i must be maximizing over all $G_i \in (0, R_i]$ (resp., $G_i \in [0, R_i]$), its payoff $V_i(G_i, G_j) \equiv \{[0.5(1-\pi)(1-\sigma)(B_1+B_2)] + [\pi.B_i + (1-\pi)[G_i^{\gamma} /(G_1^{\gamma}+G_2^{\gamma})].\sigma.(B_1+B_2)]\}$, where $[G_k + B_k] = R_k$ for $k = 1, 2$. Now, any 'interior equilibrium' $\{G_1^{\dagger}, G_2^{\dagger}\} \in (0, R_1)\times(0, R_2)$ must simultaneously solve: $\partial V_1(G_1^{\dagger}, G_2^{\dagger})/\partial G_1 = 0$ and $\partial V_2(G_2^{\dagger}, G_1^{\dagger})/\partial G_2 = 0$. Solving this system of equations, we obtain the unique *candidate* 'interior equilibrium': $G_1^{\dagger}(\sigma, \pi, R, \rho \mid \gamma) = G_2^{\dagger}(\sigma, \pi, R, \rho \mid \gamma) = \tilde{\rho}(\pi \mid \sigma\gamma).R$, where: $\tilde{\rho}(\pi \mid \sigma\gamma) \equiv 0.5\{[(1-\pi)\sigma\gamma] / [1 + \pi + (1-\pi)\sigma\gamma]\}$ ($< 0.25 < \rho$, since $\sigma\gamma < 1$).

Is this candidate equilibrium actually an equilibrium? Keeping j's choice fixed at $G_j^{\dagger} = \tilde{\rho}(\pi \mid \sigma\gamma).R$, we want to determine whether community i can profitably deviate from $G_i^{\dagger} = \tilde{\rho}(\pi \mid \sigma\gamma).R$. It can be shown that $sign\{\partial^2 V_i(G_i, G_j) / \partial(G_i)^2\} = sign\{[B_1+B_2]\times[((\gamma-1)/G_i) - (2\gamma G_i^{(\gamma-1)} / (G_1^{\gamma}+G_2^{\gamma}))] - 2\}$. For $\gamma \leq 1$, the expression in the second curly brackets is strictly negative and so $V_i(G_i, G_j^{\dagger})$ is strictly concave in G_i and hence the candidate equilibrium is an equilibrium. Next, we consider the case when $\gamma \in (1, 1/\sigma)$. Given $[B_1+B_2] > 0$, it is straightforward to check that the second expression in curly brackets is strictly positive when G_i is close to 0 and it can change sign a maximum of one time as G_i is increased from 0 up to R_i, keeping $G_j > 0$ fixed. So, keeping $G_j > 0$ fixed, either $V_i(G_i, G_j)$ is convex for all G_i or it goes from being convex when G_i is close to 0 to being concave at larger values of G_i. Further, it can be verified that $sign\{\partial^2 V_i(G_i, G_j = G_i)/\partial(G_i)^2\} = (-)$. [So, $G_i = \tilde{\rho}(\pi \mid \sigma\gamma).R$ is a local maxima for $V_i(G_i, \tilde{\rho}(\pi \mid \sigma\gamma).R)$.] Hence, the only deviation by community i from the

[3] It is easy to verify that this expression also provides community i's final payoff when $B_j = 0$ and $g_i = 0$.

candidate equilibrium $\{G_1^\dagger, G_2^\dagger\} = \{[\tilde{\rho}(\pi\,|\,\sigma\gamma).R], [\tilde{\rho}(\pi\,|\,\sigma\gamma).R]\}$ that we need to check is the deviation of G_i to an 'arbitrarily small but positive' υ (since i's payoff $V_i(G_i, G_j^\dagger)$ falls discontinuously at $G_i = 0$ (since: $B_j^\dagger > 0$), a deviation to $G_i = 0$ will be strictly dominated by a deviation to $G_i = \upsilon$). Given $\gamma < 2$ (since we have: $\gamma < 1/\sigma$ and $\sigma \in [½, 1)$), it is easy to verify that $V_i(\tilde{\rho}(\pi\,|\,\sigma\gamma).R, \tilde{\rho}(\pi\,|\,\sigma\gamma).R) > V_i(\upsilon, \tilde{\rho}(\pi\,|\,\sigma\gamma).R)$, and so no profitable deviation from the candidate equilibrium exists. Hence, $G_1^\dagger(\sigma, \pi, R, \rho\,|\,\gamma) = G_2^\dagger(\sigma, \pi, R, \rho\,|\,\gamma) = \tilde{\rho}(\pi\,|\,\sigma\gamma).R$ is indeed an equilibrium. But is it the unique equilibrium?

We next establish that there cannot exist any corner equilibrium. The only corner equilibria that could possibly exist are those where: $G_i^\dagger = R_i$ and $G_j^\dagger \in (0, R_j)$. Given $\rho > 0.25$ and $\gamma < 1/\sigma$, it can be shown through fairly straightforward algebra that when community j optimally responds to its conjectured $G_i = R_i$, then taking that response as community i's conjecture i's best response is *not* $G_i = R_i$. Hence, there exists no corner equilibrium.

So, the unique SPNE payoffs for communities 1 and 2 are determined by setting $W_i^\dagger(\sigma, \pi, R, \rho\,|\,\gamma) = V_i(\tilde{\rho}(\pi\,|\,\sigma\gamma).R, \tilde{\rho}(\pi\,|\,\sigma\gamma).R)$, and they are respectively given, for all $\pi \in [0, 1)$, as:

$$W_1^\dagger(\sigma, \pi, R, \rho\,|\,\gamma) = \left\{\pi(1-\rho) + \frac{[1-\pi][1+(1-\sigma\gamma)\pi]}{2[1+\sigma\gamma+(1-\sigma\gamma)\pi]}\right\}.R,$$

$$W_2^\dagger(\sigma, \pi, R, \rho\,|\,\gamma) = \left\{\pi\rho + \frac{[1-\pi][1+(1-\sigma\gamma)\pi]}{2[1+\sigma\gamma+(1-\sigma\gamma)\pi]}\right\}.R.$$

Notice that the above expressions are essentially the same as those given in Proposition 2 (for the case when ρ is "large") with $\sigma < 1$ simply being replaced by $[\sigma\gamma] < 1$. Hence, the comparative statics for $W_i^\dagger(\pi\,|\,\gamma)$ w.r.t. π are similarly established, and give us that:

Community 1 will strictly prefer perfect property rights $\forall\rho > 0.25$; while community 2 will strictly prefer anarchy $\forall\rho \in (0.25, \rho^-(\sigma\gamma)]$, strictly prefer $\tilde{\pi}(\rho\,|\,\sigma\gamma) \in (0, 1)$ $\forall\rho \in (\rho^-(\sigma\gamma), \rho^+(\sigma\gamma))$, and strictly prefer perfect property rights $\forall\rho \in [\rho^+(\sigma\gamma), 0.5)$. □

PROOF OF LEMMA 7:

Given feasible structural parameters including a well behaved *RLM*, let the inheritance vector in generation t be $\{R_1^*(t), R_2^*(t)\}$ with $R^*(t) \equiv [R_1^*(t) + R_2^*(t)] > 0$ and $\rho^*(t) \equiv R_2^*(t)/R^*(t) \in (0, ½)$. The two communities in generation t aim to myopically maximize their own current consumptions. For any choice of $\pi(t)$, the immediate implication of Proposition 2 is that $B_1^*(t) > B_2^*(t)$ and $G_1^*(t) \geq G_2^*(t)$. Then the fact that $W_1^*(t) \geq W_2^*(t)$ follows since each community i's final consumption $W_i^*(t)$ is {half of net bargaining surplus + own outside option} $\equiv$ $\{[0.5(1-\pi(t))(1-\sigma)(B_1^*(t) + B_2^*(t))] + [\pi(t).B_i^*(t) + (1-\pi(t))[G_i^*(t)/(G_1^*(t) + G_2^*(t))]\sigma(B_1^*(t) + B_2^*(t))]\}$. Then, as the proof of Proposition 4 clarifies, the equilibrium consumption of each community will be single-peaked in property rights, and so, for each community there will be a unique optimal level of property rights in the inherited rights space. In the rights negotiation game, community 2 will be able to establish its uniquely optimal choice. In this way, the generation t outcome vector will be uniquely determined as a function of generation t's inheritance vector. The unique generation t outcome vector will then determine the unique inheritance vector of generation t+1, in which the rights negotiation space will be a compact subset of [0, 1] by construction, and (given the well-behaved *RLM*) it will be the case that $R_1^*(t+1) > R_2^*(t+1)$ since $R_1^*(t) > R_2^*(t)$, $B_1^*(t) > B_2^*(t)$ and $W_1^*(t) \geq W_2^*(t)$. □

PROOF OF FOOTNOTE 53:

For the first two *RLM* structures, i.e., the non-linear and non-separable ones, we will be considering the equilibrium induced in the guns-and-butter game in any single generation given resource endowments $R_1 > R_2 > 0$ (or equivalently, given: $R \equiv [R_1 + R_2] > 0$ and $\rho \equiv R_2/R \in (0, 0.5)$) and *established* property rights π in the generation. To conveniently denote the guns-and-butter equilibrium quantities in the generation, we will use the simpler notation: (B_i, W_i) for $i = 1, 2$ (specifically, while representing the equilibrium quantities, we will suppress the use of "daggers" and the dependence on various arguments). Using the resource endowments and the guns-and-butter equilibrium quantities in the current generation, we aim to determine the properties of resource inequality bequeathed to the next generation in the continuation equilibrium of the game.

First, consider the following non-linear generalization of the linear homogeneous *RLM*: $F(R_i, B_i, W_i) = [x.(R_i)^{\chi} + y.(B_i)^{\chi} + z.(W_i)^{\chi}]$ with $\chi > 0$, $x \geq 1$, $y > 0$, $z > 0$.

Suppose it is the case that $\pi < 1$ and $\rho \in (\tilde{\rho}(\pi\,|\,\sigma), 0.5)$. The numerator of the derivative of $[y.(B_2)^{\chi} + z.(W_2)^{\chi}] / [y.(B_1)^{\chi} + z.(W_1)^{\chi}]$ w.r.t. χ is (with the denominator of the derivative simply being $[y.(B_1)^{\chi} + z.(W_1)^{\chi}]^2 > 0$): $[y.(B_1)^{\chi} + z.(W_1)^{\chi}].[y.ln(B_2).(B_2)^{\chi} + z.ln(W_2).(W_2)^{\chi}] - [y.(B_2)^{\chi} + z.(W_2)^{\chi}].[y.ln(B_1).(B_1)^{\chi} + z.ln(W_1).(W_1)^{\chi}] = y^2.(B_1)^{\chi}.(B_2)^{\chi}.ln(B_2/B_1) + y.z.(B_1)^{\chi}.(W_2)^{\chi}.ln(W_2/B_1) + y.z.(B_2)^{\chi}.(W_1)^{\chi}.ln(B_2/W_1) + z^2.(W_1)^{\chi}.(W_2)^{\chi}.ln(W_2/W_1)$. It is easy to verify the following equilibrium relationships: $B_2 < W_2 \leq W_1 < B_1$, and so: $(B_2/B_1) < 1$, $(W_2/B_1) < 1$, $(B_2/W_1) < 1$ and $(W_2/W_1) \leq 1$. Hence, the derivative of $[y.(B_2)^{\chi} + z.(W_2)^{\chi}] / [y.(B_1)^{\chi} + z.(W_1)^{\chi}]$ w.r.t. χ is negative and so $[y.(B_2)^{\chi} + z.(W_2)^{\chi}] / [y.(B_1)^{\chi} + z.(W_1)^{\chi}]$ is decreasing in χ. First, consider the case where $\{y/z > (2-\sigma)/\sigma\}$: With the linear homogeneous *RLM*, i.e., with $\chi = 1$, when $\{y/z > (2-\sigma)/\sigma\}$, and given $\pi < 1$ and $\rho \in (\tilde{\rho}(\pi\,|\,\sigma), 0.5)$, then we had: $[y.(B_2)^{\chi} + z.(W_2)^{\chi}] / [y.(B_1)^{\chi} + z.(W_1)^{\chi}] < R_2/R_1$, hence this is also true for $\chi > 1$ (since $[y.(B_2)^{\chi} + z.(W_2)^{\chi}] / [y.(B_1)^{\chi} + z.(W_1)^{\chi}]$ is decreasing in χ). Further, $[x.(R_2)^{\chi}]/[x.(R_1)^{\chi}] < R_2/R_1$ when $\chi > 1$. So, we have that: $[x.(R_2)^{\chi} + y.(B_2)^{\chi} + z.(W_2)^{\chi}] / [x.(R_1)^{\chi} + y.(B_1)^{\chi} + z.(W_1)^{\chi}] < R_2/R_1$ when $\chi > 1$. Second, consider the case where $\{y/z < 1/\sigma\}$: With the linear homogeneous *RLM*, i.e., with $\chi = 1$, when $\{y/z < 1/\sigma\}$, and given $\pi < 1$ and $\rho \in (\tilde{\rho}(\pi\,|\,\sigma), 0.5)$, then we had: $[y.(B_2)^{\chi} + z.(W_2)^{\chi}] / [y.(B_1)^{\chi} + z.(W_1)^{\chi}] > R_2/R_1$, hence this is also true for $\chi < 1$ (since $[y.(B_2)^{\chi} + z.(W_2)^{\chi}] / [y.(B_1)^{\chi} + z.(W_1)^{\chi}]$ is decreasing in χ). Further, $[x.(R_2)^{\chi}]/[x.(R_1)^{\chi}] > R_2/R_1$ when $\chi < 1$. So, we have that: $[x.(R_2)^{\chi} + y.(B_2)^{\chi}$

$+ z.(W_2)^\chi] / [x.(R_1)^\chi + y.(B_1)^\chi + z.(W_1)^\chi] > R_2/R_1$ when $\chi < 1$ (This is similarly seen to be true for the specific case when: $\pi < 1$ and $\rho = \tilde{\rho}(\pi | \sigma)$).

Next, suppose it is the case that $\pi < 1$ and $\rho \leq \tilde{\rho}(\pi | \sigma)$. The numerator of the derivative of $[x.(R_2)^\chi + y.(B_2)^\chi + z.(W_2)^\chi] / [x.(R_1)^\chi + y.(B_1)^\chi + z.(W_1)^\chi]$ w.r.t. χ is (with the denominator of the derivative simply being $[x.(R_1)^\chi + y.(B_1)^\chi + z.(W_1)^\chi]^2 > 0$): $[x.(R_1)^\chi + y.(B_1)^\chi + z.(W_1)^\chi].[x.ln(R_2).(R_2)^\chi + y.ln(B_2).(B_2)^\chi + z.ln(W_2).(W_2)^\chi] - [x.(R_2)^\chi + y.(B_2)^\chi + z.(W_2)^\chi].[x.ln(R_1).(R_1)^\chi + y.ln(B_1).(B_1)^\chi + z.ln(W_1).(W_1)^\chi] = x^2.(R_1)^\chi.(R_2)^\chi.ln(R_2/R_1) + x.y.(R_1)^\chi.(B_2)^\chi.ln(B_2/R_1) + x.z.(R_1)^\chi.(W_2)^\chi.ln(W_2/R_1)$ + (6 more similar terms). It is easy to verify the following equilibrium relationships: $0 = B_2 < R_2 < W_2 \leq W_1 < B_1 < R_1$, and so: $(R_2/R_1) < 1$, $(B_2/R_1) < 1$, $(W_2/R_1) < 1$, $(R_2/B_1) < 1$, $(B_2/B_1) < 1$, $(W_2/B_1) < 1$, $(R_2/W_1) < 1$, $(B_2/W_1) < 1$ and $(W_2/W_1) \leq 1$. Hence, the derivative of $[x.(R_2)^\chi + y.(B_2)^\chi + z.(W_2)^\chi] / [x.(R_1)^\chi + y.(B_1)^\chi + z.(W_1)^\chi]$ w.r.t. χ is negative and so $[x.(R_2)^\chi + y.(B_2)^\chi + z.(W_2)^\chi] / [x.(R_1)^\chi + y.(B_1)^\chi + z.(W_1)^\chi]$ is decreasing in χ. First, consider the case where $\{y/z > (2-\sigma)/\sigma\}$: With the linear homogeneous *RLM*, i.e., with $\chi = 1$, when $\{y/z > (2-\sigma)/\sigma\}$, and given $\pi < 1$ and $\rho \leq \tilde{\rho}(\pi | \sigma)$, then we had: $[x.(R_2)^\chi + y.(B_2)^\chi + z.(W_2)^\chi] / [x.(R_1)^\chi + y.(B_1)^\chi + z.(W_1)^\chi] < \tilde{\rho}(\pi | \sigma)/(1-\tilde{\rho}(\pi | \sigma))$, hence this is also true for $\chi > 1$ (since $[x.(R_2)^\chi + y.(B_2)^\chi + z.(W_2)^\chi] / [x.(R_1)^\chi + y.(B_1)^\chi + z.(W_1)^\chi]$ is decreasing in χ). Second, consider the case where $\{y/z < 1/\sigma\}$: With the linear homogeneous *RLM*, i.e., with $\chi = 1$, when $\{y/z < 1/\sigma\}$, and given $\pi = 0$ and $\rho \leq \tilde{\rho}(\pi = 0 | \sigma)$, then we had: $[x.(R_2)^\chi + y.(B_2)^\chi + z.(W_2)^\chi] / [x.(R_1)^\chi + y.(B_1)^\chi + z.(W_1)^\chi] > R_2/R_1$, hence this is also true for $\chi < 1$ (since $[x.(R_2)^\chi + y.(B_2)^\chi + z.(W_2)^\chi] / [x.(R_1)^\chi + y.(B_1)^\chi + z.(W_1)^\chi]$ is decreasing in χ).

Next, suppose it is the case that $\pi = 1$ and $\rho \geq \rho^+(\sigma)$. We get: $[x.(R_2)^\chi + y.(B_2)^\chi + z.(W_2)^\chi] / [x.(R_1)^\chi + y.(B_1)^\chi + z.(W_1)^\chi] = [(x + y + z).(R_2)^\chi] / [(x + y + z).(R_1)^\chi] = (R_2)^\chi/(R_1)^\chi$. For $\chi > 1$, $(R_2)^\chi/(R_1)^\chi < (R_2)/(R_1)$ and for $\chi < 1$, $(R_2)^\chi/(R_1)^\chi > (R_2)/(R_1)$.

Hence, when $\{y/z > (2-\sigma)/\sigma\}$, then for $\chi > 1$ the *RLM* satisfies Property F-1.2, and so using Proposition 10 we have that the equilibrium path $\{\pi^*(t), \rho^*(t)\}_{t=1}^{\infty}$ devolves to anarchy for all $\rho(1) \in (0, 0.5)$. Alternatively, in the case where $\{y/z < 1/\sigma\}$, for $\chi < 1$ the *RLM* satisfies Property F-2, and so using Proposition 10 we have that the equilibrium path $\{\pi^*(t), \rho^*(t)\}_{t=1}^{\infty}$ attains perfect property rights for all $\rho(1) \in (0, 0.5)$.

Second, consider the non-separable *RLM*s. Consider: $F(R_i, B_i, W_i) = [x.R_i^{(\chi_1+\chi_2)} + y.R_i^{\chi_1}.B_i^{\chi_2}]$ with $\{x \geq 1, y > 0, \chi_1 > 0, \chi_2 > 0\}$ and $[\chi_1 + \chi_2] > 1$. We have for all $\pi \in [0, 1]$ and $\rho \in (0, 0.5)$ that: $B_2/B_1 \leq R_2/R_1$, and $R_2^{(\chi_1+\chi_2)}/R_1^{(\chi_1+\chi_2)} < R_2/R_1$ since $[\chi_1 + \chi_2] > 1$, so $[x.R_2^{(\chi_1+\chi_2)} + y.R_2^{\chi_1}.B_2^{\chi_2}] / [x.R_1^{(\chi_1+\chi_2)} + y.R_1^{\chi_1}.B_1^{\chi_2}] < R_2/R_1$. Hence, the homogeneous *RLM* satisfies Property F-1.2, and so the equilibrium path $\{\pi^*(t), \rho^*(t)\}_{t=1}^{\infty}$ devolves to anarchy for all $\rho(1) \in (0, 0.5)$. Next, consider: $F(R_i, B_i, W_i) = [x.R_i^{(\chi_1+\chi_3)} + z.R_i^{\chi_1}.W_i^{\chi_3}]$ with $\{x \geq 1, z > 0, \chi_1 > 0, \chi_3 > 0\}$ and $[\chi_1 + \chi_3] \leq 1$. We have for all $\pi \in [0, 1]$ and $\rho \in (0, 0.5)$ that: $W_2/W_1 \geq R_2/R_1$, and $R_2^{(\chi_1+\chi_3)}/R_1^{(\chi_1+\chi_3)} > R_2/R_1$ since $[\chi_1 + \chi_3] < 1$, so $[x.R_2^{(\chi_1+\chi_3)} + z.R_2^{\chi_1}.W_2^{\chi_3}] / [x.R_1^{(\chi_1+\chi_3)} +$

$z.R_1^{\chi_1}.W_1^{\chi_3}$] > R_2/R_1. Hence, the homogeneous *RLM* satisfies Property F-2, and so the equilibrium path $\{\pi^*(t), \rho^*(t)\}_{t=1^\infty}$ attains perfect property rights for all $\rho(1) \in (0, 0.5)$.

Third, consider the following time-varying linear *RLM*: $F(R_i, B_i, W_i; (\rho, \pi)) = x(\rho, \pi).R_i + y(\rho, \pi).B_i + z(\rho, \pi).W_i$, where $x(.) \geq 1$, $y(.) > 0$ and $z(.) > 0$ are continuous functions of their arguments (and where $\rho \in (0, 0.5]$ and $\pi \in [0, 1]$). It is straightforward to see that starting from $R_1(1) > R_2(1) > 0$ and some initial property rights environment, a unique equilibrium time-path $\{\pi^*(t), \rho^*(t)\}_{t=1^\infty}$ will be induced with the property that: $R_1^*(t) > R_2^*(t) > 0$ for all t (and hence: $R^*(t) \equiv [R_1^*(t) + R_2^*(t)] > 0$ and $\rho^*(t) \equiv R_2^*(t)/R^*(t) \in (0, 0.5)$ for all t). Given inherited resources $R_1 > R_2 > 0$ (or equivalently, given: $R \equiv [R_1 + R_2] > 0$ and $\rho \equiv R_2/R \in (0, 0.5)$) and *established* property rights π in a generation, let us denote the resource inequality parameter inherited by the next generation (in the continuation equilibrium of the game) as: $r^\dagger(\sigma, \pi, R, \rho) \equiv F(\rho R, B_2^\dagger(\sigma, \pi, R, \rho), W_2^\dagger(\sigma, \pi, R, \rho); (\rho, \pi)) / [F((1-\rho)R, B_1^\dagger(\sigma, \pi, R, \rho), W_1^\dagger(\sigma, \pi, R, \rho); (\rho, \pi)) + F(\rho R, B_2^\dagger(\sigma, \pi, R, \rho), W_2^\dagger(\sigma, \pi, R, \rho); (\rho, \pi))]$. Consider the case when: $\{y(\rho, \pi)/z(\rho, \pi) > (2-\sigma)/\sigma\}$ for all ρ and π. We cannot directly invoke Propositions 10 and 11 here since the *RLM* is not just a function of R_i, B_i and W_i. In any generation in which established rights $\pi < 1$ and $\rho \in (\tilde{\rho}(\pi | \sigma), 0.5)$, since $\{y(\rho, \pi)/z(\rho, \pi) > (2-\sigma)/\sigma\}$ so, as we established in the proof of proposition 11, we will get: $r^\dagger(\sigma, \pi, R, \rho) < \rho$. In any generation in which established rights $\pi < 1$ and $\rho \leq \tilde{\rho}(\pi | \sigma)$, since $\{y(\rho, \pi)/z(\rho, \pi) > (2-\sigma)/\sigma\}$ so, as we established in the proof of proposition 11, we will get: $r^\dagger(\sigma, \pi, R, \rho) < \tilde{\rho}(\pi | \sigma)$. So, if the equilibrium path is such that $\rho^*(t) \leq 0.5\sigma/(1+\sigma)$ in some generation t, then resource inequality will remain there for all later generations. We will next establish (like we did in the proof of Proposition 10) that the equilibrium path, starting from $\rho(1) \in (0.5\sigma/(1+\sigma), \rho^+(\sigma))$, does not converge weakly above $0.5\sigma/(1+\sigma)$. We will achieve this by assuming the contrary hypothesis and then establishing a contradiction: Suppose that $\{\pi^*(t), \rho^*(t)\}_{t=1^\infty}$ converges to $\rho' \in [0.5\sigma/(1+\sigma), \rho(1))$; then, $\rho^*(t)$ will come arbitrarily close to ρ' for large t (since resource inequality will keep strictly increasing over time), and eventually (i.e., for all large t): $\pi^*(t) = \pi^\dagger(\rho^*(t) | \sigma)$. Define the following function of $\rho \in [0.5\sigma/(1+\sigma), \rho^+(\sigma))$ given $\sigma \in [0.5, 1)$ and $R > 0$: $J(\rho | \sigma) \equiv r^\dagger(\sigma, \pi^\dagger(\rho | \sigma), R, \rho) < \rho$. Note that all the R's will get cancelled and so $J(\rho | \sigma)$ is independent of R. It is easily shown (as in the proof of Proposition 10) that $J(\rho | \sigma)$ is continuous over its domain. Then, using the fact that for all large t we have $\pi^*(t) = \pi^\dagger(\rho^*(t) | \sigma)$, we get that for all large t: $\rho^*(t+1) = J(\rho^*(t) | \sigma)$. Using this fact and the fact that $\rho^*(t)$ converges to ρ', we can show that the function $J(\rho | \sigma)$ is discontinuous at $\rho = \rho'$, which is a contradiction to the fact that the function is continuous over its domain. Hence, our assumption that $\{\pi^*(t), \rho^*(t)\}_{t=1^\infty}$ converges to $\rho' \geq 0.5\sigma/(1+\sigma)$ was incorrect, and so for all large enough t we will get: $\rho^*(t) \leq 0.5\sigma/(1+\sigma)$. Since $0.5\sigma/(1+\sigma) < \rho^-(\sigma)$, so using Proposition 8 we get that the equilibrium path $\{\pi^*(t), \rho^*(t)\}_{t=1^\infty}$ devolves to anarchy for all $\rho(1) \in (0, \rho^+(\sigma))$. (It is straightforward to check the proof of

Proposition 8 to see that we do not require the *RLM* to be 'well-behaved' or for it to only depend on R_i, B_i and W_i for that result to hold. Having already established that $R_1*(t) > R_2*(t) > 0$ for all t, we can directly invoke Proposition 8 here.) The case when $\{y(\rho, \pi)/z(\rho, \pi) < 1/\sigma\}$ for all ρ and π is handled using similar arguments and it can be shown that the equilibrium path $\{\pi^*(t), \rho^*(t)\}_{t=1}^{\infty}$ attains perfect property rights for all $\rho(1) \in (0, 0.5)$. □